

Using Distributed Fiber-optic Strain Sensing to Estimate Generalized Modal Coordinates from Flight-test Data

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Background and Motivation

Problem

- Contemporary aircraft carry around structural mass so that the flutter instabilities lie well outside of the operational envelope.
- Better methods of measuring the structural state could allow reduction of the extra structural weight

Modal filtering

- Standard method in structural analysis
- Deformations are a combination of mode shapes
- Modal filtering is estimating these modal coordinates from data
 - Often ordinary least squares methods
 - Often applied to simpler test articles

Factor Analysis

- Analysis method from psychology
- Measurements are a combination of small number of unmeasurable variables.
- Lessons from factor analysis can be adapted to improve the modal filtering methods

Method

Math Model

Fitting Method

- Factor Model
- Relating Shapes to the FEM
- Estimating Generalized Coordinates

Results

- Maneuver Description
- Simulated Data
- Flight Data

Definition of Model

Factor Model

$$\mathbf{z} = \Phi \boldsymbol{\eta} + \boldsymbol{\nu}$$

- Measurements (\mathbf{z}) are a linear combination of modal generalized coordinates ($\boldsymbol{\eta}$), measurement noise ($\boldsymbol{\nu}$)
 - Only looking at sensor equation
 - No process noise like turbulence
- Noise is assumed to be independent gaussian white noise

Covariance of the outputs

$$\boldsymbol{\Sigma} = \Phi \boldsymbol{\kappa} \mathbf{P} \boldsymbol{\kappa} \Phi^T + \mathbf{V}$$

- As a result of gaussian assumptions, the covariance ($\boldsymbol{\Sigma}$) is a Wishart random variable
- Fitting is finding the parameters that maximize the Wishart likelihood

$$\mathcal{L} = n \cdot \log |\boldsymbol{\Sigma}| + \text{tr}(\boldsymbol{\Sigma} \mathbf{Z} \mathbf{Z}^T) + \text{constant}$$

- Generalized coordinates are not in the covariance matrix

Maximum likelihood estimation of factor model

Estimating:

- Modes shapes
- Variance of modal coordinates
- Correlation of modal coordinates
- Sensor noise variance

Not all parameters will always be estimated

- Mode shapes can come from the FEM
- Assume that noise is the same for all sensors

Relating Shapes to the FEM

Shapes can be pulled from FEM

- FEM shapes have errors
- May not capture the data well

Estimated shapes can be rotated

- How are shapes related back to the FEM shapes?
- Least-squares rotation
- Procrustes rotation





Estimating generalized modal coordinates

Ordinary least squares (OLS)

$$\hat{\boldsymbol{\eta}}^{ols} = (\boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^T \mathbf{Z}$$

- Coordinates that minimize the sensor error
- Common classical approach

Bartlett (weighted least squares, WLS)

$$\hat{\boldsymbol{\eta}}^{wls} = (\boldsymbol{\kappa} \boldsymbol{\Phi}^T \mathbf{V}^{-1} \boldsymbol{\Phi} \boldsymbol{\kappa})^{-1} \boldsymbol{\kappa} \boldsymbol{\Phi}^T \mathbf{V}^{-1} \mathbf{Z}$$

- Coordinates that minimize the sensor error normalized by noise
- Estimates biased larger by noise

Thompson (regression, REG)

$$\hat{\boldsymbol{\eta}}^{reg} = \left[(\boldsymbol{\kappa} \boldsymbol{\Phi}^T \mathbf{V}^{-1} \boldsymbol{\Phi} \boldsymbol{\kappa})^{-1} + \mathbf{P} \right] \hat{\boldsymbol{\eta}}^{wls}$$

- Maximum likelihood estimate of coordinates given measurements
- Consistent with a Kalman filter

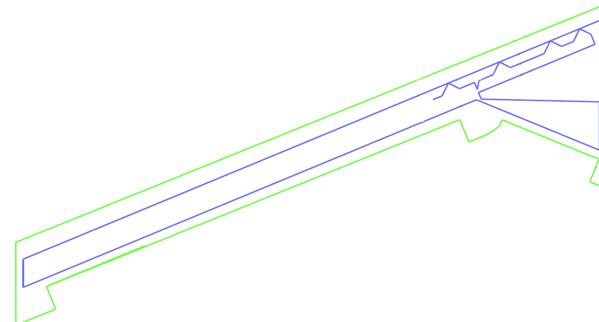
Flight Test Results



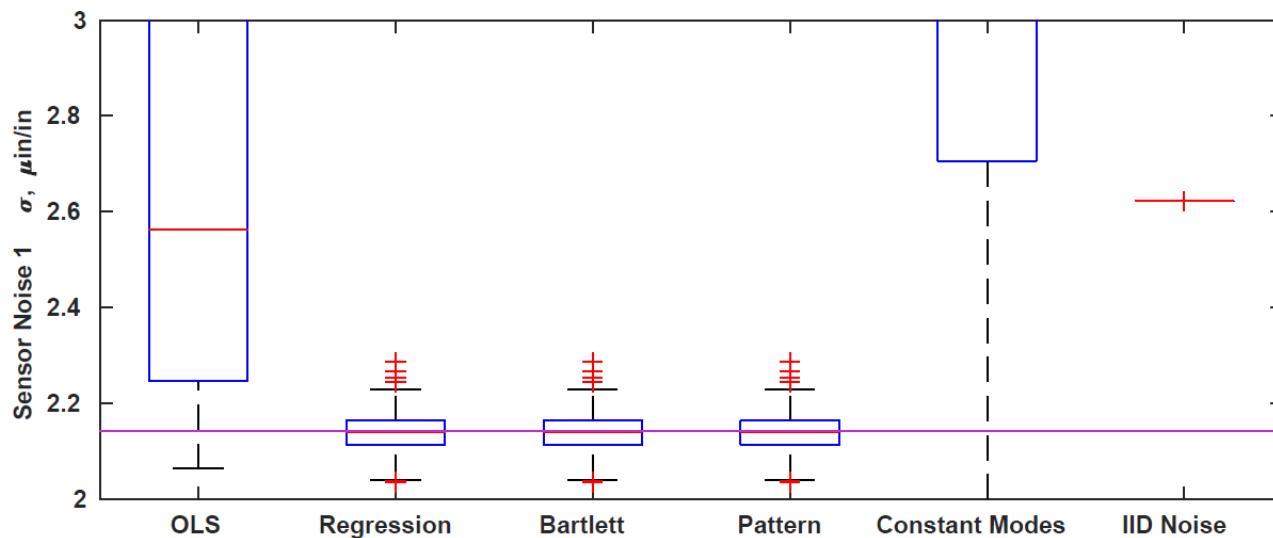
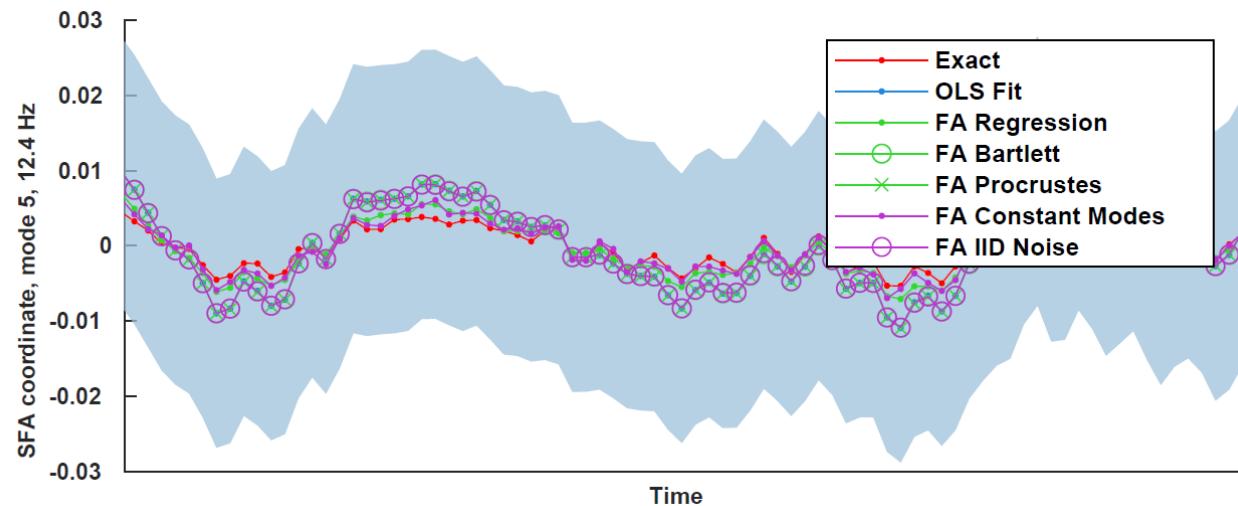
Case	Airspeed, kn	Fuel, lb	Flight
1	80	62	16
2	110	56	26

X-56A flex wing
Symmetric multisines
Above and below flutter speed
Simulated and flight data
936 strain measurements

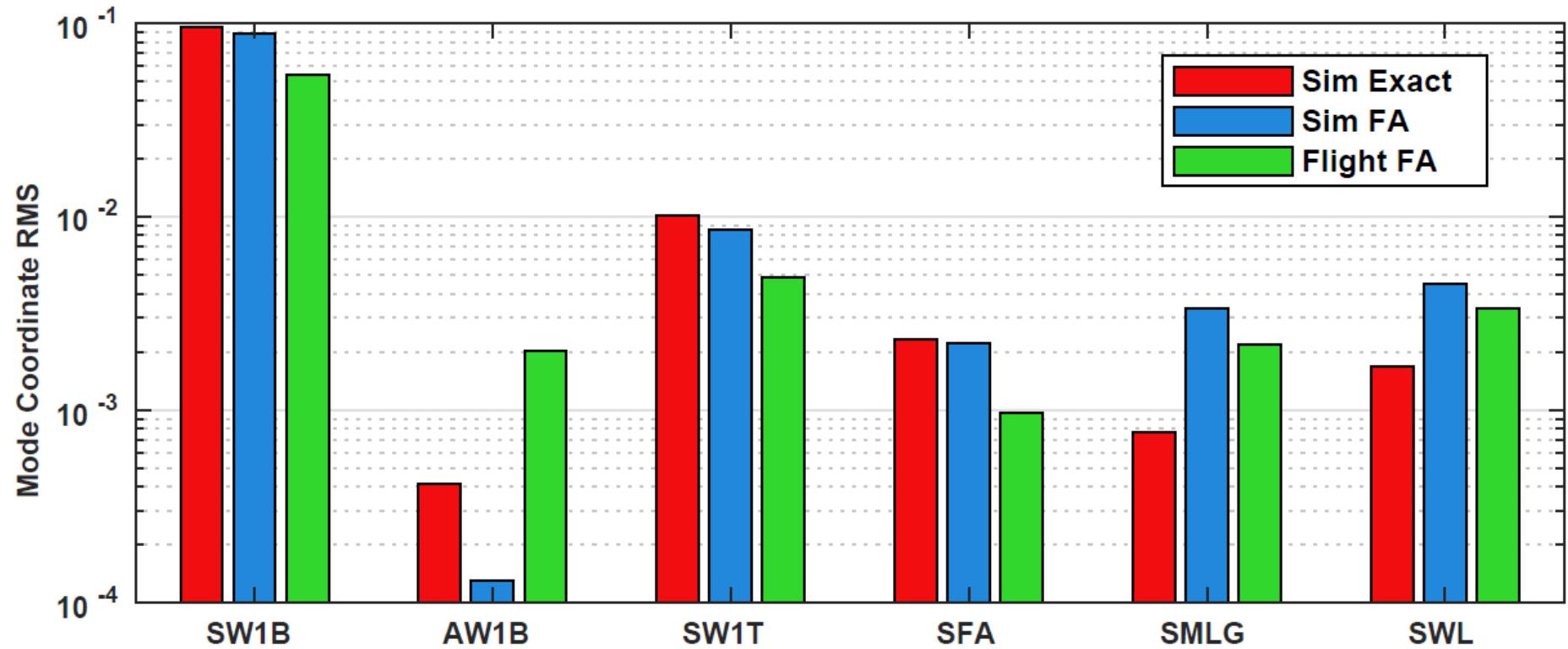
Case	Mode Shapes Estimator	Rotation	Coordinate Estimator	Sensor Noise
Exact	N/A	N/A	N/A	N/A
OLS	Constant	N/A	OLS	Free
FA Regression	Varying	Procrustes	Regression	Free
FA Bartlett	Varying	Procrustes	Bartlett	Free
FA Pattern	Varying	Pattern	Regression	Free
FA Constant	Constant	N/A	Regression	Free
FA IID Noise	Constant	N/A	Regression	Uniform



Simulated Data



Flight Data



Conclusion

Maximum Likelihood estimator of statistics of modal coordinates

- Mode shapes
- Noise

Statistics used to inform estimates of modal coordinates

Accuracy of estimators compared for simulation and flight data

- Each estimator had strengths and weaknesses
- Least square rotation matches mode shapes well, but gives poor state estimates
- FEM mode shapes gave good estimates of the states, but poor estimates of the noise
- The regression estimator with Procrustes rotation generally gave the best results